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# Quantum critical point in the spin glass–antiferromagnetism competition in Kondo lattice systems

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## Abstract

A theory is proposed to describe the competition among antiferromagnetism (AF), spin glasses (SG) and the Kondo effect. The model describes two Kondo sublattices with an intrasite Kondo interaction strength  $J_K$  and an interlattice quantum Ising interaction in the presence of a transverse field  $\Gamma$ . The interlattice coupling is a random Gaussian distributed variable (with average  $-2J_0/N$  and variance  $32J^2/N$ ) while the  $\Gamma$  field is introduced as a quantum mechanism to produce spin flipping. The path integral formalism is used to study this fermionic problem where the spin operators are represented by bilinear combinations of Grassmann fields. The disorder is treated within the framework of the replica trick. The free energy and the order parameters of the problem are obtained by using the static ansatz and by choosing both  $J_0/J$  and  $\Gamma/J \approx (J_K/J)^2$  to allow, as previously, a better comparison with the experimental findings. The results indicate the presence of a SG solution at low  $J_K/J$  and for temperature  $T < T_f$  ( $T_f$  is the freezing temperature). When  $J_K/J$  is increased, a mixed phase AF + SG appears, then an AF solution and finally a Kondo state is obtained for high values of  $J_K/J$ . Moreover, the behaviours of the freezing and Néel temperatures are also affected by the relationship between  $J_K$  and the transverse field  $\Gamma$ . The first presents a slight decrease while the second decreases towards a quantum critical point (QCP). The obtained phase diagram has the same sequence as the experimental one for  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$ , if  $J_K$  is assumed to increase with  $x$ , and in addition it also shows a qualitative agreement concerning the behaviour of the freezing and the Néel temperatures.

## 1. Introduction

The competition between the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction and the Kondo effect is recognized as the fundamental mechanism for explaining several properties of Ce and U compounds. Nevertheless, there is also strong experimental evidence that the presence of disorder in those compounds can be source of quite novel effects [1–5]. For instance, deviations of the Fermi liquid (FL) behaviour are observed in the magnetic, thermodynamic and transport properties. Recently, theories have been proposed to explain how disorder can produce such deviations [6, 7]. However, it is still not clear whether disorder itself can be the origin of such deviations. Another perspective has adopted the proximity to a QCP [8] as a possible origin of the non-Fermi liquid (NFL) behaviour. As well as this complicated scenario with NFL behaviour, some of these physical systems present a highly non-trivial manifestation of disorder, which is frustration. As a consequence, there are many phase diagrams with ferromagnetism (FM), antiferromagnetism (AF), spin glass (SG) and a Kondo state with the complete screening of the localized spins. This raises the question of whether it is possible to construct theories able to account for those phase diagrams.

The experimental situation for Ce and U alloys, which present together a spin glass phase, an antiferromagnetic phase and finally a Kondo phase is complicated, especially when the phase changes occur when changing the matrix composition of the alloy. Moreover, there is certainly an important local effect in the spin glass, as recently evidenced by local measurements in  $\text{CeCu}_{1-x}\text{Ni}_x$  alloys [9]. In particular, there is a change from AF to Kondo and then to SG with increasing  $x$  in  $\text{UCu}_{5-x}\text{Pd}_x$  [10] and a different situation in  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys [11]. More precisely, in the case of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys, the glassy behaviour is not favoured when the chemically induced disorder is initially increased. In the experimental phase diagram, for  $0 < x < 0.45$  and low temperature, a spin glass-like state appears. When the Co doping is increased, the system experiences a transition to an AF phase. In the interval  $0.45 < x < 0.9$ , the Néel temperature  $T_N$  decreases until a QCP is reached at  $x = 0.9$  with no trace of NFL behaviour. For  $x > 0.9$ , the magnetic moments are suppressed due to the Kondo effect.

Recently, a theoretical approach [12] has attempted to describe the interplay between AF and SG in a Kondo lattice using the same framework previously introduced to study the SG in the Kondo lattice [13]. The proposed model is a two-Kondo sublattice with an intrasite exchange interaction with strength  $J_K$  and a random Gaussian  $J_{ij}$  intersite interaction only between distinct sublattices. Two quite important points are introduced in this approach. The first one is that there is no hopping of conduction electrons between distinct sublattices. Therefore, the antiferromagnetic solutions in this model are entirely associated with the coupling among localized magnetic moments. The second one takes the relationship  $J_0/J \propto (J_K/J)^2$ , because the two exchange terms cannot be considered as completely independent from each other [16]. As a consequence, when the strength of the intrasite exchange interaction  $J_K$  increases (in units of  $J$ ), the degree of frustration  $J/J_0$  decreases. This last parameter controls the emergence of SG or AF solutions in the problem. The results have been shown in a phase diagram of  $T/J$  ( $T$  is the temperature) versus  $J_K/J$ . For small  $J_K/J$  and low temperature there is a SG phase. When  $J_K/J$  is increased, an AF solution has been found. Finally, for  $J_K/J \geq 15$ , the Kondo state becomes dominant. Thus, we can conclude from the previous calculation [12] that the sequence of phases SG, AF and Kondo mimics qualitatively the experimental phase diagram of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  if  $J_K/J$  is associated with the Co content.

However, we can say that the previous agreement was only qualitative to provide the correct sequence of phases for the alloys  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  [11], because in the previous approach there was no mechanism able to produce a QCP. As a consequence, this approach

cannot provide the proper behaviour of the Néel temperature which should decrease towards a QCP. Thus, the purpose of the present paper is to obtain specifically a quantitative description of the phase diagram of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys, and in particular of the QCP, by adding a quantum mechanism to produce spin flipping, given by an additional transverse field  $\Gamma$  [17].

We take here the replica symmetry mean-field approximation and we use the static approximation. Indeed, such an approximation is subject to criticism, but it has been shown in many previous works that it yields a good description of the competition between the Kondo effect and magnetism in the Kondo lattice models. Moreover, the quantum Ising spin glass in a transverse field has the same critical behaviour as the  $M$ -component quantum rotor which is exactly solvable in the  $M \rightarrow \infty$  limit [18]. In this model, the line transition is given by the singularity of the zero-frequency that is equivalent to the static approximation. The static approximation is, therefore, relatively well justified for the study of phase boundaries, which is really the purpose of our paper in the specific case of the SG–AF–Kondo competition in alloys such as  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$ .

Therefore, we will assume a relationship  $\Gamma \propto J_K^2$ ; this choice, which has already been used in [16, 17], is taken to account for the fact that the intrasite exchange interaction is able to produce both the Kondo effect and the RKKY interaction. The transverse field plays a role similar to the spin flipping part of the Heisenberg model. One of the main achievements of this approach has been to show the presence of a QCP in the competition between SG and the Kondo state when  $\Gamma$  is enhanced and therefore to considerably improve the description of experimental results.

In this paper, we study the SG–AF competition in the disordered Kondo lattice, described by the hopping of the conduction electrons only inside each sublattice, an intersite exchange interaction, given by an Ising-like term only between different sublattices, and a transverse field  $\Gamma$  applied in the  $x$ -direction, in order to obtain a mechanism able to produce a QCP. We use also the static approximation and the relationships  $J_0 \propto J_K^2$  and  $\Gamma \propto J_K^2$  already introduced in [12, 17]. Therefore, the initial many-parameter problem is reduced to one parameter which allows a better comparison with experimental results.

The comparison of our theoretical results to experimental data is delicate for two reasons. First, as we have just discussed, we take a  $J_K^2$  dependence of both  $J_0$  and  $\Gamma$ , which is clearly a crude approximation but which has already given interesting physical results in [16]. Then, the second question concerns the fact that  $J_K$  is assumed to increase with the concentration of Co in  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys, as previously done with the concentration of Ni in  $\text{CeCu}_{1-x}\text{Ni}_x$  alloys. Indeed, the role of the so-called ‘chemical pressure’ is less clear than that of the regular pressure, but obviously the spin glass phase originates in such alloys only from the disorder of the matrix and we are obliged to make an assumption of the relationship between  $J_K$  and the relative host concentration  $x$ , which was previously successful in the case of  $\text{CeCu}_{1-x}\text{Ni}_x$  alloys [17]. This question is in fact not easy to answer theoretically.

In section 2, we introduce the model and the relevant order parameters for the problem. The saddle point free energy in terms of the order parameters is obtained so that it allows us to obtain a phase diagram giving temperature  $T/J$  versus  $J_K/J$ . The results and the final discussion are presented in the last section.

## 2. General formulation

We have considered in this work a model given by two Kondo sublattices  $A$  and  $B$  with a random coupling  $J_{ij}$  only between localized spins in distinct sublattices [12, 19]. There is also a transverse field  $\Gamma$  coupled with the localized spins in both sublattices [17]. As mentioned in section 1, the hopping of conduction electrons between two different sublattices is not allowed

for simplicity. Therefore, the Hamiltonian is:

$$H - \mu N = \sum_{p=A,B} \left[ \sum_{i,j} \sum_{\sigma=\uparrow\downarrow} t_{ij} \hat{d}_{i,p,\sigma}^\dagger \hat{d}_{j,p,\sigma} + \sum_i \varepsilon_{0,p}^f \hat{n}_{i,p}^f \right. \\ \left. + J_K \left( \sum_i \hat{S}_{i,p}^+ \hat{S}_{i,p}^- + \hat{S}_{i,p}^- \hat{S}_{i,p}^+ \right) \right] + \sum_{i,j} J_{ij} \hat{S}_{i,A}^z \hat{S}_{j,B}^z + 2\Gamma \sum_i (\hat{S}_{i,A}^x + \hat{S}_{i,B}^x) \quad (1)$$

where  $i$  and  $j$  sums are over  $N$  sites of each sublattice. The intersite coupling  $J_{ij}$  is a random variable following a Gaussian distribution with average  $-2J_0/N$  and variance  $32J^2/N$ . The spin operators present in equation (1) are defined as in [12]:  $\hat{S}_{i,p}^+ = \hat{d}_{i,p,\uparrow}^\dagger \hat{d}_{i,p,\downarrow} = (\hat{S}_{i,p}^-)^\dagger$ ,  $\hat{S}_{i,p}^- = \hat{f}_{i,p,\uparrow}^\dagger \hat{f}_{i,p,\downarrow} = (\hat{S}_{i,p}^+)^\dagger$ ,  $\hat{S}_{i,p}^z = \frac{1}{2} [\hat{f}_{i,p,\uparrow}^\dagger \hat{f}_{i,p,\uparrow} - \hat{f}_{i,p,\downarrow}^\dagger \hat{f}_{i,p,\downarrow}]$  and  $\hat{S}_{i,p}^x = \frac{1}{2} [\hat{f}_{i,p,\uparrow}^\dagger \hat{f}_{i,p,\downarrow} + \hat{f}_{i,p,\downarrow}^\dagger \hat{f}_{i,p,\uparrow}]$  where  $\hat{d}_{i,p,\sigma}^\dagger$ ,  $\hat{d}_{i,p,\sigma}$  ( $\hat{f}_{i,p,\sigma}^\dagger$ ,  $\hat{f}_{i,p,\sigma}$ ) are the creation and destruction operators for conduction (localized) fermions.

The partition function is given in the Lagrangian path integral formalism in terms of Grassmann variables [12, 13]  $\psi_{i,p,\sigma}(\tau)$  for the localized fermions and  $\varphi_{i,p,\sigma}(\tau)$  for the conduction ones. The Kondo interaction is treated following [12]. The Kondo state is described by complex order parameters  $\lambda_{p,\sigma} = \frac{1}{N} \sum_{i,\sigma} \langle \varphi_{i,p,\sigma}^* \psi_{i,p,\sigma} \rangle$  which are introduced in the partition function using the integral representation of delta function. Therefore, the partition function when  $\lambda_{p\sigma} \approx \lambda_p$  becomes:

$$Z/Z_0^d = \exp\{-2N\beta J_K (|\lambda_A|^2 + |\lambda_B|^2)\} \int \prod_{p=A,B} D(\psi_p^* \psi_p) \exp \left[ A_{\text{eff}}^{(\text{stat})} \right] \quad (2)$$

where  $Z_0^d$  is the partition function of the free conduction electrons,

$$A_{\text{eff}}^{(\text{stat})} = \sum_{i,j} \sum_{\omega} \underline{\Theta}_i^\dagger(\omega) \left[ \underline{g}_{ij}(\omega) \right]^{-1} \underline{\Theta}_j(\omega) + \beta \sum_{ij} J_{ij} S_{i,A}^z S_{j,B}^z, \quad (3)$$

$$\underline{\Theta}_i^\dagger(\omega) = [\psi_{i,A,\uparrow}^*(\omega) \psi_{i,A,\downarrow}^*(\omega) \psi_{i,B,\uparrow}^*(\omega) \psi_{i,B,\downarrow}^*(\omega)] \quad (4)$$

and  $[\underline{g}_{ij}(\omega)]^{-1}$  is a  $4 \times 4$  matrix given by

$$\left[ \underline{g}_{ij}(\omega) \right]^{-1} = \begin{bmatrix} \underline{F}_{A,ij}(\omega) & \underline{0} \\ \underline{0} & \underline{F}_{B,ij}(\omega) \end{bmatrix} \quad (5)$$

with

$$\underline{F}_{p,ij}(\omega) = \begin{bmatrix} (i\omega - \beta\varepsilon_{0,p}^f) & \beta\Gamma \\ \beta\Gamma & (i\omega - \beta\varepsilon_{0,p}^f) \end{bmatrix} \delta_{i,j} - \sum_K \frac{\beta^2 J_K^2 |\lambda_p|^2 e^{i\vec{k}(\vec{R}_i - \vec{R}_j)}}{(i\omega - \beta\varepsilon_0) - \beta\varepsilon_K} \underline{I}_p \quad (6)$$

The notations  $\underline{I}$  and  $\underline{0}$  represent the identity and the null  $2 \times 2$  matrices, respectively.

The free energy is given by the replica method. The procedures are quite close to reference [12] and in particular to its appendix, but now with the presence of the transverse field  $\Gamma$  which modifies the matrix  $[\underline{g}_{ij}(\omega)]^{-1}$ . Therefore, the averaged free energy within the replica symmetric theory can be obtained as

$$\beta F = \beta J_K (|\lambda_A|^2 + |\lambda_B|^2) - \beta^2 \frac{J^2}{2} (q_A q_B - \tilde{q}_A \tilde{q}_B) - \beta \frac{J_0}{2} m_A m_B - \lim_{n \rightarrow 0} \frac{1}{2Nn} \\ \times \ln \int_{-\infty}^{\infty} \prod_{i=1}^N D\xi_{i,A} D\xi_{i,B} \prod_{\alpha} \int_{-\infty}^{\infty} Dz_{i,A}^\alpha Dz_{j,B}^\alpha \\ \times \exp \left[ \sum_{w,\sigma} \ln \det \left[ \underline{G}_{ij}(\omega) h_{i,p}^\alpha \right]^{-1} \right] \quad (7)$$

where the matrix  $[\underline{G}_{ij}(\omega|h_{i,p}^\alpha)]^{-1}$  in equation (7) is the same matrix given in equation (5), except that the matrix  $\underline{F}_{ij,p}(\omega)$  (see equation (6)) is replaced by  $\underline{F}_{ij,p}(\omega) + \sigma\beta J h_{i,p}^\alpha \delta_{ij} \underline{I}$  ( $\sigma = +(\uparrow), -(\downarrow)$ ). In equation (7)  $Dy = dy \exp(-y^2/2)/\sqrt{2\pi}$  ( $y = \xi, z$ ). The internal field  $h_{i,p}^\alpha$  applied in the sublattice  $p = A, B$  is defined as

$$h_{i,p}^\alpha = J\sqrt{2q_{p'}}\xi_{i,p} + J\sqrt{2\chi_{p'}}z_{i,p}^\alpha - J_0 m_{p'}. \quad (8)$$

In equation (8),  $q_{p'}$ ,  $\chi_{p'} = \beta\bar{\chi}_{p'}$  and  $m_{p'}$  are the SG order parameter, the static susceptibility and the magnetization, respectively. It should be noticed that the internal field applied in sublattice  $p$  depends on the behaviour of  $q_{p'}$ ,  $\chi_{p'}$  and  $m_{p'}$  with  $p' \neq p$ . That is a direct consequence of the choice made in the model where only interlattice frustration has been considered in the present work.

Finally, to solve equation (7), the matrix  $[\underline{G}_{ij}(\omega|h_{i,p}^\alpha)]^{-1}$  is substituted by  $[\underline{G}_{\mu\nu}(\omega|h_{i,p}^\alpha)]^{-1}$ . In fact, this approximation [12, 13, 17] represents a Kondo sublattice  $p$  in the presence of a constant magnetic field  $h_{i,p}^\alpha$  where the effects of the Kondo sublattice  $p'$  are placed. Therefore, in equation (7), we have:

$$\ln \det [\underline{G}_{ij}(\omega|h_{i,p}^\alpha)]^{-1} = \frac{1}{N} \sum_j \ln \det [\underline{G}_{\mu\nu}(\omega|h_{i,p}^\alpha)]^{-1}. \quad (9)$$

The sum over the Matsubara frequencies in equation (7) can now be performed as usual [12, 13, 17]. The proposed decoupling also allows us to use Fourier transformation which can be evaluated by assuming a constant density of states for the conduction electron band  $\rho(\epsilon) = 1/2D$  for  $-D < \epsilon < D$ . Thereby, the free energy is obtained as:

$$\begin{aligned} \beta F = & \beta J_K(|\lambda_A|^2 + |\lambda_B|^2) + \frac{\beta^2 J^2}{2} \bar{\chi}_A \bar{\chi}_B + \frac{\beta^2 J^2}{2} (\bar{\chi}_A q_B + \bar{\chi}_B q_A) \\ & - \frac{\beta J_0}{2} m_A m_B - \frac{1}{2} \int_{-\infty}^{\infty} D\xi_{i_A} \int_{-\infty}^{\infty} D\xi_{j_B} \ln \left[ \prod_{p=A,B} \int_{-\infty}^{\infty} Dz_p e^{E(H_p)} \right] \end{aligned} \quad (10)$$

where

$$E(H_p) = \frac{1}{\beta D} \int_{-\beta D}^{+\beta D} dx \ln \left\{ \cosh \left( \frac{x + \beta H_p}{2} \right) + \cosh(\sqrt{\Delta}) \right\} \quad (11)$$

with

$$\Delta = \frac{1}{4}(x - \beta H_p)^2 + (\beta J_K \lambda_p)^2 \quad (12)$$

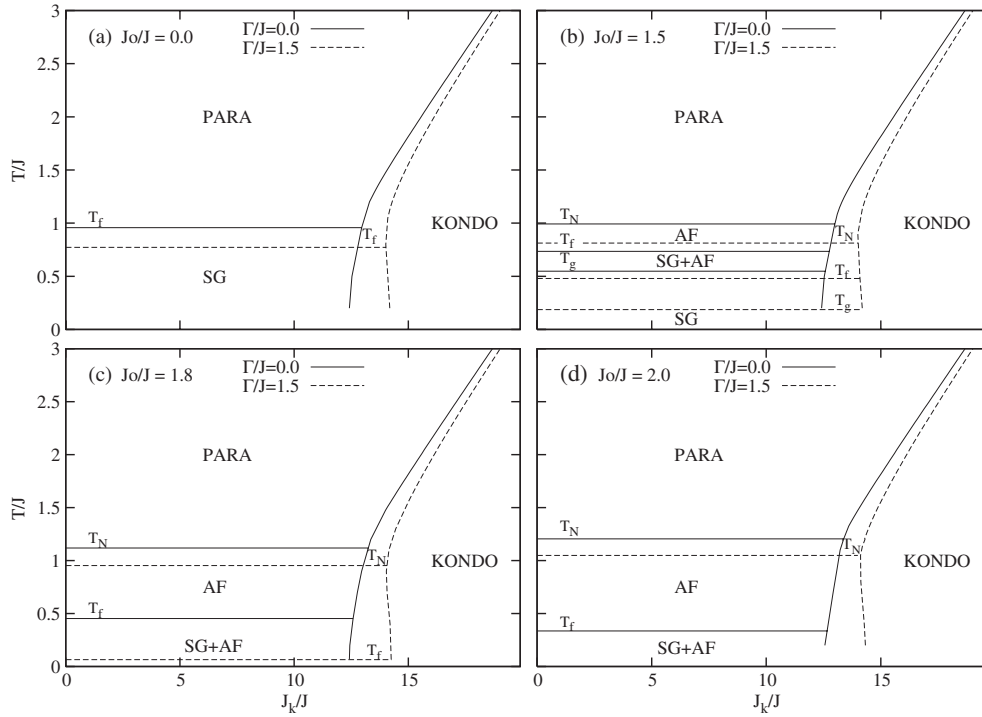
and  $H_p = \sqrt{\Gamma^2 + h_p^2}$ , with the internal field  $h_p$  given by equation (8) (here without the site and replica indices). The saddle point order parameters follow directly from equations (10)–(12).

The limit of stability for the order parameters with replica symmetry is given when the Almeida–Thouless eigenvalue  $\lambda_{AT}$  becomes negative [14, 15]:

$$\lambda_{AT} = 1 - 2(\beta J)^4 \prod_{p=A,B} \int_{-\infty}^{\infty} D\xi_p \left[ \frac{I_p(\xi_p)}{(\int_{-\infty}^{\infty} Dz_p e^{E(H_p)})^2} \right]^2 \quad (13)$$

where

$$I_p(\xi_p) = \int_{-\infty}^{\infty} Dz_p e^{E(H_p)} \int_{-\infty}^{\infty} Dz_p \frac{\partial}{\partial h_p} \left[ e^{E(H_p)} \frac{\partial E(H_p)}{\partial h_p} \right] - \left[ \int_{-\infty}^{\infty} Dz_p e^{E(H_p)} \frac{\partial E(H_p)}{\partial h_p} \right]^2. \quad (14)$$



**Figure 1.** Theoretical phase diagram  $T/J$  versus  $J_K/J$  for several values of  $J_0/J$ : (a)  $J_0/J = 0$ , (b)  $J_0/J = 1.5$ , (c)  $J_0/J = 1.8$  and (d)  $J_0/J = 2.0$ . The full lines correspond to the results obtained for  $\Gamma/J = 0.0$  while the dashed ones are for  $\Gamma/J = 1.5$ .

### 3. Results and conclusions

The central issue of the present work is to account for the general features of the experimental phase diagram of Ce alloys such as  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  [11] which present with increasing Co concentration the successive sequence of the SG phase and then the AF phase with the Néel temperature  $T_N$  decreasing towards a QCP. In principle, the solutions for the saddle point order parameters  $q_p$ ,  $\bar{\chi}_p$ ,  $m_p$  and  $\lambda_p$  ( $p = A, B$ ) should be found in a parameter space with axes given by  $J_0/J$ ,  $J_K/J$  and  $\Gamma/J$  ( $J$  and  $D$  are kept constant).

Therefore, in order to identify the role of each parameter in the problem, the solutions for  $q_p$ ,  $\bar{\chi}_p$ ,  $m_p$  and  $\lambda_p$  ( $p = A, B$ ) are shown in figure 1 as a phase diagram  $T/J$  versus  $J_K/J$  for several values of  $J_0/J$  and  $\Gamma/J$ . The SG solution corresponds to  $q_A = q_B \neq 0$  (with  $m_A = m_B = 0$ ) while the AF solution is equivalent to  $m_A = -m_B \neq 0$  and  $q_A = q_B \neq 0$ . The Kondo state in the phase diagram corresponds to the situation where the two  $\lambda_p$  ( $p = A, B$ ) are different from zero. In figure 1(a),  $J_0 = 0$  and  $\Gamma = 0$  or  $\Gamma/J = 1.5$ , this situation corresponds to the studied case in [17] in which there is no AF solution. For  $\Gamma = 0$ , the phase diagram displays three solutions. At high temperature and small  $J_K/J$ , a paramagnetic (PARA) solution is found. When the temperature is decreased, there is a transition to the SG phase at  $T_g$ , which coincides with the  $T_f$  temperature that locates the Almeida–Thouless line (the temperature where the Almeida–Thouless eigenvalue is zero). If  $J_K/J$  is enhanced, there is a transition at  $J_K = J_{K_1}(T/J)$  to a region where the Kondo solution is dominant. When  $\Gamma/J$  is tuned on, there is a clear effect on the transition lines  $T_g$  and  $J_{K_1}(T/J)$ ; the first one is decreased while the second is displaced to a larger  $J_K/J$  value.

When  $J_0/J$  is enhanced, as is shown in figures 1(b)–(d), an AF solution for the order parameters appears below the Néel temperature  $T_N$  for small  $J_K/J$ . For this particular situation, the set of temperatures  $T_N$ ,  $T_f$  and  $T_g$  is a function of  $\Gamma/J$  and  $J_0/J$ . For  $\Gamma = 0$ , the temperature  $T_N$  increases with  $J_0/J$  while the replica symmetric transition temperature  $T_g$  and the Almeida–Thouless line  $T_f$  decrease with  $J_0/J$ . Nevertheless, similar to [15],  $T_f$  becomes larger than  $T_g$ . Therefore, in such circumstances, there is the onset of a mixed phase AF + SG at  $T_f$ . This mixed phase represents a replica symmetry breaking SG phase with non-zero spontaneous sublattice magnetization. Finally, at  $T_g$  there is a transition to a pure SG phase. For increasing  $J_K/J$ , a Kondo solution at  $J_K > J_{K_1}(T/J)$  is found again. When the transverse field  $\Gamma/J$  is tuned on, its effect is basically to decrease simultaneously  $T_N$ ,  $T_f$  and  $T_g$  and displace the transition line  $J_{K_1}(T/J)$  as before.

However, to compare the results obtained in this work with the experimental results, it is better to reduce the number of parameters which are in fact not independent of each other, because both the Kondo effect and the RKKY interaction can be generated directly from the intrasite exchange interaction. Thus, we choose a relationship among the parameters  $J_0/J$ ,  $J_K/J$  and  $\Gamma/J$  similar to that used in [12] where a relationship  $J_0/J \propto (J_K/J)^2$  has successfully reproduced the experimental sequence of phases. Nevertheless, in [12] we cannot explain at all the decrease of the Néel temperature  $T_N$  towards a QCP. In the present work, the transverse field  $\Gamma$  has been introduced to provide a simple mechanism able to reproduce the spin flipping part of the Heisenberg model [17]. Therefore, it would also be natural to assume  $\Gamma/J \propto (J_K/J)^2$  in the present problem. It should be remarked that this choice introduces an additional complexity. It has been shown that  $\Gamma/J$  and  $J_0/J$  enforce different effects in a previously studied competition between AF and SG. Within a certain range of  $J_0/J$ , this new choice can favour SG, SG + AF or AF, while  $\Gamma/J$  tends to destroy the three phases leading the transition temperatures to zero [20].

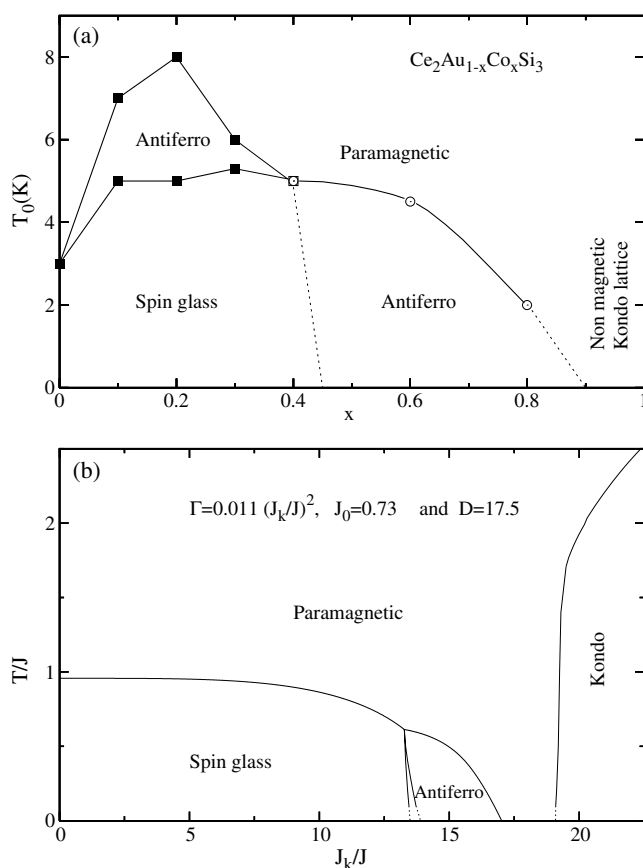
The solution of the set  $q_p$ ,  $\bar{\chi}$ ,  $m_p$  and  $\lambda_p$  with  $J_0/J \propto \Gamma/J \propto (J_K/J)^2$  is shown in figure 2(b) giving the phase diagram  $T/J$  versus  $J_K/J$ . In particular, the Néel temperature  $T_N \equiv T_N(J_K)$  can be computed expanding the saddle point equations in powers of  $m_p$ . At the second-order critical line  $T_N$ , we can make  $q_p = \lambda_p = 0$ , therefore  $\bar{\chi} = \bar{\chi}_p = 1/(\beta_c J_0)$  and

$$m \left[ 1 - \beta_c J_0 \left( \frac{1 + \int D\xi \xi^2 \cosh \beta_c \sqrt{\frac{2J\xi^2}{J_0\beta_c} + \Gamma^2}}{1 + \int D\xi \cosh \beta_c \sqrt{\frac{2J\xi^2}{J_0\beta_c} + \Gamma^2}} \right) \right] = 0, \quad (15)$$

where  $\beta_c = 1/T_N$  and  $m = m_A = -m_B$ . For  $T_N$  tending to zero, the critical value of  $\Gamma_c$  is given by analytical solution of equation (15):  $\Gamma_c = J_0 + 2J^2/J_0$ . For  $0 \leq J_K/J \leq 13.5$ , figure 2(b) shows the presence of the SG solution in which  $T_g$  and  $T_f$  coincide. Then, in a very small interval of  $J_K/J$ ,  $T_f \neq T_g$ . Therefore, the mixed phase AF + SG appears after the SG phase. Finally, in the interval  $13.9 \leq J_K/J \leq 17.0$ , only the AF solution is present. When  $J_K/J$  increases, the Néel temperature decreases until a QCP. Above this point onwards, there is a transition line to the Kondo state characterized by  $\lambda_A = \lambda_B \neq 0$  [17]. At high temperature, the solutions of the order parameters are  $q_p = 0$ ,  $m_p = 0$  and  $\lambda_p = 0$  ( $p = A, B$ ).

The situation described in the present replica symmetry mean field theory can be analysed as follows: for a low  $J_K/J$  value (corresponding to a high value of  $J/J_0$ ), frustration is dominant and the long range internal field is given by the SG component of  $h_p$  (see equation (8)). When  $J_K/J$  is increased, the degree of frustration  $J/J_0$  is decreased, and therefore the localized spin of the sublattice  $A$  ( $B$ ) starts to couple with an internal field which depends on the negative magnetization of the sublattice  $B$  ( $A$ ) in the AF ordering [12]. However, this process also implies the increase of the transverse field  $\Gamma$  and consequently the freezing temperature ( $T_g$ ) is slightly decreasing. On the other hand, the Néel temperature ( $T_N$ )





**Figure 2.** Comparison between experimental and theoretical phase diagrams: the upper figure is an experimental phase diagram of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys [11]; the lower figure is a theoretical phase diagram  $T/J$  versus  $J_K/J$  for the relations  $\Gamma = 0.011(J_K/J)^2$  and  $J_0/J = 0.73\Gamma/J$ , where the dotted lines are the extrapolations carried to lower temperature.

is deeply affected and decreases towards a QCP. Finally, for sufficiently high  $J_K/J$ , there is complete dominance of the Kondo effect [12, 13, 17]. Finally, as we have already explained in the introduction, the comparison between theory and experiment is more delicate in the case of disordered alloys, where the different phases occur as a function of the relative matrix concentration, than in the classical case, where increasing pressure makes  $J_K$  increase. But here we consider that  $J_K$  increases with increasing Co concentration, as we have successfully assumed that  $J_K$  increases with increasing Ni concentration in the case of  $\text{CeCu}_{1-x}\text{Ni}_x$  alloys [17]. Under this assumption, the theoretical results given in figure 2(b) agree quite well with the experimental phase diagram of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys [11] (see figure 2(a)) and in particular we obtain the same order for the sequence of phases and, mainly, the correct behaviour of the Néel temperature.

To conclude, we have used a two-sublattice model with two exchange interactions, an intrasite exchange and a random intersite between localized Ising spins in the presence of a transverse field. The main goal of the present work has been to reproduce some fundamental aspects of the phase boundary contained in the experimental phase diagram of the  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys [11]. Finally, the theoretical results shown in figure 2(b) account

quite well for the most important part of the experimental phase diagram of  $\text{Ce}_2\text{Au}_{1-x}\text{Co}_x\text{Si}_3$  alloys [11] with the cobalt concentration (figure 2(a)) and the agreement between the two figures showing a clear improvement with the present model in comparison with previous ones. In fact, important questions are not really solved, such as for example the different possible Kondo–AF–SG sequences found in different disordered alloys, the relationship between the parameters of the model and the matrix concentration, which is certainly less clear than the pressure dependence, and finally the precise local nature of the spin glass phase. Further work will therefore be necessary to improve the theoretical description and to obtain new phase diagrams of Ce or U alloys or compounds with either the matrix concentration or the external pressure.

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